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BOOK REVIEW

Turbulent Combustion. By Norbert Peters. Cambridge University Press, 2000. 320 pp. ISBN 0521 60823. £45.

According to the dust jacket this series of monographs exists for the publication of major new works of scholarship on all aspects of theoretical and applied mechanics. We are also told that this book is written for researchers and students of engineering and applied mathematics and aims to provide a thorough introduction to turbulent combustion within a unified presentation of the field. There are four chapters. The first gives a general introduction while the other three deal respectively with premixed, non-premixed and partially premixed turbulent combustion. They are followed by a short Epilogue, a brief glossary of common abbreviations, an extensive and up-to-date bibliography and author and subject indexes. The Preface suggests that a practical aim of the work is the development of theoretical models that can be used to reduce air pollution by improving the efficiency of combustion systems. The aim of providing a unified presentation is addressed mainly in terms of the pioneering work of Professor Peters and his colleagues at RWTH Aachen in developing the level-set approach and thin flamelet models for turbulent combustion. Some prior knowledge of turbulent flow theory and of combustion is assumed.

The first chapter has the title *Turbulent combustion: The state of the art*. It begins with a discussion of scale separation and Reynolds number independence in turbulent combustion at high Reynolds numbers and goes on to provide a concise review of statistical descriptions of turbulent flows and related matters. Balance equations are provided for reactive scalars, the formal mathematical description of chemical reaction rates is presented and methods for the systematic reduction of reaction rate mechanisms are reviewed. After discussing these and related matters, the chapter provides brief introductory sketches of current modelling approaches including, for example, a review of probability density function (p.d.f.) transport equation methods, an introduction to the laminar flamelet concept, the conditional moment closure method, and a discussion of combustion models for large-eddy simulation. The level-set function G and the mixture fraction Z, which play such important roles in the next two chapters, are introduced.

Chapter 2, which deals with premixed turbulent combustion, may be seen as the core of the whole work. The first quarter of the chapter is taken up with preliminaries: governing equations, burning velocities, experimental apparatus, controlling processes and turbulent combustion regimes are discussed. Two of these, identified as the corrugated flamelets regime and the thin reaction zones regime, are to be described in detail by the level-set method. Previous theoretical models are reviewed. The remaining three-quarters of the chapter provide a detailed and generally clear account of the level-set or G-equation approach to premixed laminar and turbulent combustion, as developed by Professor Peters and his colleagues. In this approach a scalar field $G(\boldsymbol{x}, t)$ is defined such that the condition $G(\boldsymbol{x}, t) = G_0$ is satisfied on the flame surface, and a kinematic statement of the normal propagation of this surface at a speed S_L leads to the formulation of a differential equation for G. At points where $G(\boldsymbol{x}, t) < G_0$ the mixture is unreacted and reaction is complete when $G(\boldsymbol{x}, t) > G_0$.

The advantages claimed for this approach in the context of turbulent combustion are that, when the G-equation is averaged, the conventional problems of averaging a highly nonlinear chemical source term and modelling a turbulent transport term are replaced by the need to average the flame propagation term

$$S_L|\nabla G|$$

while the turbulent transport term is absent from the mean G-equation. Conceptual difficulties are also identified. In particular it is pointed out that G is only defined at the flame surface and the consequences of this non-uniqueness are discussed in some detail for both laminar and turbulent flow models. Two different *ansatz* for $G(\boldsymbol{x}, t)$ are shown to give the same solution for a laminar Bunsen flame. In turbulent flow the p.d.f. $P(G; \boldsymbol{x}, t)$ can be formulated but it is only $P(G_0; \boldsymbol{x}, t)$ that is physically relevant as the probability of finding flame surface at $\boldsymbol{x}$ and t. Analogously, the mean and variance of $G(\boldsymbol{x}, t)$ can be written, transport equations for them can be derived and closure hypotheses introduced, but only mean quantities conditional on the presence of the flame surface have a physical meaning. This distinction between conditional and unconditional mean quantities arises, for example, in connection with the variance of G, in some modelled terms in the equations, and also in the mean velocity field where the unconditional mean velocity is not necessarily the same as the conditional mean

$$\langle u_k(\boldsymbol{x}, t) | G(\boldsymbol{x}, t) = G_0 \rangle$$

which influences the motion of the flame surface. Also, errors arise because the flow field outside the mean flame surface convects iso-scalar surfaces at mean speeds different from that of the flame surface. These errors are corrected by a numerical process referred to as *reinitialization.*

Making use of ideas of Reynolds number independence and scale separation between the small scales at which chemical reactions occur and the larger scales in the turbulent energy cascade, two different models are developed: one for the corrugated flamelets regime and the other for the thin reaction zones regime. Order-of-magnitude arguments are then used to devise a more general model for application in both regimes. An important assumption, used to guide model development, is that the mean G-equation for turbulent flow must have similar structure and mathematical properties to the G-equation for laminar flow. This leads *inter alia* to a closure of the mean G-equation in terms of a modelled turbulent burning velocity. Effects of density change and gas expansion due to heat release are treated indirectly, by modifying the equations for the mean and variance of G.

Publication of this chapter serves a valuable purpose by bringing together material from a large number of research papers, together with some new material, and presenting it all in a generally clear and unified manner. There are places where I would have liked to understand more clearly, such as the justification at some points in the analysis for replacing a conditional mean quantity by an unconditional one, and the role of the reinitialization process. However, starting from his stated assumptions, the author has made significant progress towards the development of a rational theory, and this is an impressive achievement.

In Chapter 3, Professor Peters describes models of non-premixed turbulent combustion. Some of the material contained in this chapter will be more familiar to readers: the mixture fraction Z, the Burke–Schumann solution, other solutions involving the mixture fraction in laminar flames, a turbulent combustion regimes diagram in which laminar flamelets have an important role. A new derivation of the laminar flamelet equations is presented, involving a two-scale asymptotic analysis, followed by a description of a presumed p.d.f. flamelet model for turbulent combustion. Among other topics, there is an interesting analysis of non-unity Lewis number effects, a rather brief description of the incorporation of unsteady flamelet effects into a laminar flamelet model and a discussion of the prediction of pollutant emissions from non-premixed turbulent flames. The chapter provides a clear and concise review of this class of models.

The relatively short final chapter has the title *Partially premixed turbulent combustion.* The lifted turbulent jet flame is identified as a generic problem and the chapter contains a useful review of experimental work on this topic. The triple flame is found to be a key element in partially premixed combustion. Turbulent combustion models are reviewed and a model based on a combination of the G-equation and the Z-equation is outlined.

The two-page Epilogue emphasizes that models of turbulent combustion require two types of input: from the classical theory and modelling of non-reactive turbulent flows, and from asymptotic flame theory based on scale separation. Only leading-order results can be used when the input from both areas is to be combined.

Does this book live up to the aims set out on the dust jacket and quoted at the beginning of this review? Yes, I believe it does. It is a major new work of scholarship in an important area. This book does present a unified description based on clearly stated premises and I recommend anyone with a serious research interest in turbulent combustion to read it. Nevertheless we must recall that turbulent combustion is still a rapidly developing and often controversial topic. Other works with titles very similar to this book can give strikingly different descriptions of it. Students of the subject must still be encouraged to read widely.

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